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Abstract
The life cycle hypothesis has become the dominant mode used to analyze the effects of a social security system on private saving, the labor/leisure choice, and social welfare. As both Barro and Samuelson indicate, a fully funded Social Security program (in a world of certainty) would drive out an equivalent amount of private saving. If the interest rate is r, the effects of a payment of a dollar into the social security pool while young would just offset* the effects of receiving (1+r) dollars as a transfer when retired. Papers by Diamond, Hi», and Samuelson, among others, have examined the effects of non-fully funded Social Security schemes in a growing economy. A non-fully funded program can be used to alter the private sector's saving rate and, hence, the capital/labor ratio. Social Security, then, can be used as a policy tool for achieving the (or some variant of the) golden rule growth path.

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The life cycle hypothesis has become the dominant mode used to analyze the effects of a social security system on private saving, the labor/leisure choice, and social welfare. As both Barro and Samuelson indicate, a fully funded Social Security program (in a world of certainty) would drive out an equivalent amount of private saving. If the interest rate is r, the effects of a payment of a dollar into the social security pool while young would just offset the effects of receiving (1+r) dollars as a transfer when retired. Papers by Diamond, Hu, and Samuelson, among others, have examined the effects of non-fully funded Social Security schemes in a growing economy. A non-fully funded program can be used to alter the private sector's saving rate and, hence, the capital/labor ratio. Social Security, then, can be used as a policy tool for achieving the (or some variant of the) golden rule growth path. In this regard, an optimal Social Security scheme can be found which will increase the long-run well-being of an economy. Authors such as Feldstein and Hu have argued that Social Security can be expected to reduce the labor supply if the time of retirement is endogenous. In essence, Social Security acts as a tax on earnings for those remaining on the job after the normal retirement age.

While important, the results discussed above were derived from models which abstracted from uncertainty. It is the purpose of this paper to develop a life cycle model which highlights the role of uncertainty. The model is then used to re-examine the effects of Social Security on private saving, the labor/leisure choice, and social welfare. It is shown that in an uncertain world, a fully funded Social Security program will **not**, in general, drive out an equivalent amount of private saving. Even if the program is actuarially fair from an individual's point of view, a fully funded plan can be expected to alter the supply of labor. Within the context of our model, it is shown that--in the absence of uncertainty--private decisions are optimal; at best, a Social Security plan will have no real effects. However, we shall show that under uncertainty, private decisions will not, in general, be optimal even if individuals have rational expectations. Consequently, some scope exists for social policy to produce an improvement in social welfare, and it will be demonstrated that a Social Security scheme exists that will produce an efficient allocation of resources. Perhaps our most interesting results concern second best Social Security plans, i.e. those which distort labor decisions. It is shown that distortionary plans exist that lead to an improvement in social welfare.

At this point some caveats concerning the nature of our model are in order. It is assumed that individuals live for two periods; choosing the division of time between labor and leisure in the first, but retired in the second. Thus, as opposed to Feldstein and Hu, we do not allow for work during the second period of life. Individuals in their first period of life must make the interrelated decisions of how much to work, consume and save for the second period of life. We assume that population is constant, that output is not storable and that labor is the only factor of production. Thus, money is the only store of value, and we cannot analyze the effects of Social Security taxes on growth paths à la Diamond, Hu, and Samuelson. Lastly, we do not consider the transition during which the Social Security program is being implemented. Readers interested in such issues should see Flemming. Yet, we do not believe that these simplifications are overly severe for they serve to highlight the role of uncertainty. Uncertainty enters the model in the form of a stochastic production function disturbance. Given this exogenous output uncertainty, the overall price level and the value of ones savings for retirement are also uncertain. As stated above, in the absence of this output uncertainty, there would be no scope for a Social Security plan in the context of our model. Thus, all of our findings concerning the beneficial effects of Social Security arise because Social Security acts to reduce the undesirable effects of uncertainty. It should be stressed that the beneficial effects of Social Security plans are not due to myopia on the part of the private sector or to Ponzi/chain-letter effects. We assume that individuals have rational expectations and our assumptions of a constant population and no physical capital rule out chain-letter effects. Social Security programs act as a means to pool risk across generations. While the price mechanism does, to some extent, allow for intergenerational risk sharing, a Social Security plan can induce the socially optimal degree of risk sharing. With Social Security taxes being proportional to income, social security is a means to transfer income from high to low output generations. If risk averse individuals do not know whether they will be members of a high or low output generation, they will, in general, prefer some form of intergenerational risk sharing.

The first section of the paper develops the model and shows that in the absence of uncertainty, fully-funded Social Security plans will have--at best--no real effects. The second section of the paper presents our theoretical results concerning Social Security schemes. Some of the effects of Social Security depend upon the form of the utility function and the distribution of the exogenous output disturbance. The third section of the paper presents simulation results for various distributions of the disturbances and parameters for the class of additive - constant relative risk aversion utility functions. Conclusions and directions for further research are presented in the last section of the paper.

I. The Model

In accord with the basic life cycle approach, we assume:

(i) Individuals live for only two periods; thus, an individual of generation t is born at the beginning of t and dies at the end of period t+1.

(ii) The individual can work only in the first period of his life, but consumes in both periods.

(iii) Individual utility - for a member of generation t - is given by: $U_t = U(1-L_t, C_t^t, C_t^{t+1})$, where:

L_t = work in t; $(1-L_t)$ = leisure

C_t^j = consumption in j by a member of generation t; j=t, t+1 .

(iv) The economy is stationary; each generation consists of N identical individuals; preferences of all individuals of all generations are identical.

(v) Aggregate output is linear in total labor supply; output for each period is subject to a random disturbance. This disturbance is identical for members of the same generation. Further, the output disturbance for different generations is identically, but independently, distributed.

(vi) A Social Security plan exists whereby an individual pays a constant fraction of his earned income in taxes, and receives a transfer during retirement. More will be said on this later.

(vii) Individuals choose their labor supply and consumption decisions to maximize expected utility; furthermore, they have rational expectations, i.e. their subjective probability distributions are identical with the true probability distributions.

(viii) Money is the only store of value for individuals, as commodities are assumed perishable. Further, the money stock is constant (so that Social Security benefits in t equal tax revenues in t).

(ix) Commodity prices - and their distribution - are determined by aggregate equilibrium.

Mathematically, let:

(1) L_t^i = the labor supply of individual i of generation t,

(2) $L_t^a = \sum_{i=1}^{N} L_t^i$; aggregate labor supply at t.

(3) $Q_t = A_t L_t^a$; $E(A_t) = 1$, $\sigma^2(A_t) > 0$.

In (3), E( ) denotes the expectation operator; below we let $E_{\{x,y\}}(\ )$ denote that expectations run over x and y. Output in each period (t) is subject to a random productivity disturbance: A_t and A_{t+i} $(i \neq 0)$ are assumed identically, but independently, distributed. For individual i of generation t, net income is:

(4) $Y_t^i = P_t A_t L_t^i (1-\tau)$,

where τ is the tax rate, and P_t the money price of commodities. Each individual pays taxes in the first period of life, receiving transfers in the second; let:

(5) R_{t+1}^i = transfer to i - of generation t - at (t+1).

In general, the transfers to be received are random; assuming full-funding, i.e., no money creation, so that transfers at t equal taxes at t:

(6) $R_t^a = \tau \cdot P_t \cdot A_t L_t^a$,

where R_t^a is aggregate transfers at t to members of generation (t-1). Further, let:

(7) $C_t^a = \sum_{i=1}^{N} {}^iC_t^t$;

where ${}^iC_t^t$ is the first period consumption rule for members of generation t, and C_t^a is the aggregate consumption demand of the new generation.

At t, the old generation's aggregate buying power is $(\overline{M} + R_t^a)$, where $\overline{M}$ is the outstanding money stock[1]; thus, aggregate demand equilibrium is given by:

(8) $\left(\frac{\bar{M} + R_t^a}{P_t}\right) + C_t^a = A_t L_t^a$; or:

(9) $\frac{\bar{M}}{P_t} = A_t L_t^a\ (1-\tau) - C_t^a$

Given the aggregate labor supply, consumption demand and the realized value of A_t, (9) determines the price level (P_t); ex ante, it determines the true distribution of prices (P_t). Further, since the economy is stationary, the distributions of P_t and P_{t+i} are identical, but independent.

Returning to individuals, they must choose (L_t^i), $({}^iC_t^t)$ to maximize expected utility, given their subjective beliefs; consistent with rational expectations, their beliefs about prices are governed by (9). Further, we assume the maximization process for the individual consists of two steps:

(i) He must choose L_t^i before A_t, P_t, and future prices and transfers are known;

(ii) His consumption demand, ${}^iC_t^t$, is made after P_t and A_t are known, but before P_{t+1} and R_{t+1}^i (his net transfer next period) are known.

Given L_t^i, ${}^iC_t^t$, his second period consumption is

(10) ${}^iC_t^{t+1} = [P_t(A_t L_t^i(1-\tau) - {}^iC_t^t) + R_{t+1}^i]/P_{t+1}$

Thus, given P_t, A_t, L_t^i, the individual chooses ${}^iC_t^t$ to maximize:

$$\underset{P_{t+1}, R_{t+1}^i}{E_i}\ [U(1-\tilde{L}_t^i,\ {}^iC_t^t,\ {}^iC_t^{t+1})],$$

where the $\sim$ denotes L_t^i is predetermined. This optimization procedure yields the individual's consumption demand function, ${}^iC_t^t$, which depends on P_t, A_t, L_t^i and expectations of P_{t+1}, R_{t+1}^i. Finally, L_t^i is determined by

(11) $\text{Max} \underset{\{P_t, A_t, P_{t+1}, R_{t+1}^i\}}{E} [U(1-L_t^i,\ {}^iC_t^t,\ {}^iC_t^{t+1})]$

From (11), L_t^i is determined; finally, a rational expectations solution requires that individual beliefs reflect the actual distributions; and hence that the aggregate consumption-labor supply decisions assumed in (9) are consistent with individual decisions.

In general, it is not possible to determine analytic solutions for this problem in general form, so we propose to present results for the special case of additive, constant relative risk aversion utility functions. However, before doing this, we shall specify the two types of Social Security plans we shall analyze and discuss their impact in a world of certainty. The first plan - which we call the proportional plan - ties the benefits the individual receives to his work effort (and hence to his expected contributions):

$$\text{(i)} \quad R^i_{t+1} = \tau\, P_{t+1}\, A_{t+1}(L^i_t/L^a_t)\; L^a_{t+1}$$

The second plan - what we call the flat rebate plan or distortionary plan - gives an equal amount to all retired individuals regardless of their prior work experience[2]:

$$\text{(ii)} \quad R^i_{t+1} = \tau\, P_{t+1}\, A_{t+1}\, (L^a_{t+1}/N)$$

Note that for either plan total benefits paid out in (t+1) equal tax receipts in (t+1), as assumed earlier. Also, note that for either plan the real value of the benefits an individual will receive are random, depending on the realization of (A_{t+1}).

Under certainty $(A_t=A_{t+1}\equiv 1)$, it is clear from the stationarity of the system that P is constant over time (given a constant tax rate). Further, since all individuals are alike $L^a_{t+1}=L^a_t=L^a=N\cdot L^i$. Clearly, the proportional tax plan has no effect on individual consumption in either period or labor supply (under certainty), as can be seen directly from (10) - the plan leaves relative prices, the real return to labor, and the feasible consumption set unaltered. In essence, the government is imposing forced savings on individuals (equal to τPL^i_t); to offset this, individuals simply reduce private savings (the demand for money) by a comparable amount. Hence, the plan does not alter real variables, but does cause domestic prices to increase. This, then, is a classic case of how individual agents counteract the actions of government, as indicated by Barro and Samuelson.

On the other hand, the flat rebate scheme will affect the allocation of resources, since it decreases the real return to labor; that is, if an individual agent chooses to work an extra hour, his net first period income increases by $P(1-\tau)$, whereas his net benefits in the next period are unaltered. Assuming both goods are net substitutes for leisure, and that leisure and both good are normal, it can readily be shown that the net effect of plan (2) is to reduce labor supply and consumption in each period.[3] Consequently, this plan reduces utility for all generations and agents cannot successfully circumvent it.

Thus, we see that in a world of certainty, there is no positive role to be played by Social Security taxes within a model like ours. At best, a proportional rebate plan (scheme 1) has no real effect but does increase prices; a flat rebate plan (scheme 2) distorts decisions and leads to inefficiency. We now turn to analyze the impact of these plans under uncertainty for additive, constant relative risk aversion utility functions.

II. Impact of Social Security Schemes for Specific Utility Functions

For tractability, we assume each agent's utility function is additive and exhibits constant relative risk aversion:

$$(12)\quad U_t^i = (1/\rho)\left[(1-L_t^i)^\rho + ({}^iC_t^t)^\rho + \lambda({}^iC_t^{t+1})^\rho\right];\ \lambda\le 1;\ \rho<1;\ \rho\ne 0.$$

For $\rho = 0$, (12) reduces to :

$$(12')\quad U_t^i = \ln[1-L_t^i] + \ln[{}^iC_t^t] + \lambda\ \ln[{}^iC_t^{t+1}];\ \lambda \le 1$$

In the above equations, $\lambda(\le 1)$ represents the rate of time preference. Note that ρ measures the degree of risk aversion (as well as the degree of substitutability among commodities); specifically, $(1-\rho)$ is the (absolute value of the) degree of relative risk aversion.

A) The Logarithmic Case

We take this - the easiest case - first. Individual beliefs governing prices are given by (9). Given L_t^i, the individual chooses ${}^iC_t^t$ to maximize:

$$(12'')\quad U = \ln(1-L_t^i) + \ln\ {}^iC_t^t + \lambda \underset{\{A_{t+1}\ R_{t+1}^i\}}{E} \ln[{}^iC_t^{t+1}];\ \text{or:}$$

$$(13)\quad \frac{1}{{}^{i}C_t^t} = \lambda\ E\left[\frac{P_t}{P_{t+1}\ {}^{i}C_t^{t+1}}\right], \text{ where:}$$

$$(14)\quad {}^{i}C_t^{t+1} = [P_t(A_tL_t^i(1-\tau) - {}^{i}C_t^t) + R_{t+1}^i] / P_{t+1}$$

In (13) - (14), P_t, A_t, and L_t^i are known, the distribution of P_{t+1} is determined by (9), and that of R_{t+1}^i depends on the specific Social Security plan. (13) determines the consumption demand, ${}^{i}C_t^t\ (P_t, A_t, L_t^i; \ldots)$ given expectations. Substituting ${}^{i}C_t^t$ into (12') and maximizing expected utility over L_t^i - before P_t and A_t are known - yields:

$$(15)\quad \frac{1}{1-L_t^i} = \lambda\cdot \underset{A_t, P_t, R_{t+1}^i}{E}\left[\frac{P_t\cdot A_t(1-\tau) + \dfrac{\partial R_{t+1}^i}{\partial L_t^i}}{P_{t+1}\ {}^{i}C_t^{t+1}}\right]$$

In (15), we use the fact that:

$$(16)\quad \underset{A_t, P_t}{E}\left[\left(\frac{1}{{}^{i}C_t^t} - \lambda\ \underset{R_{t+1}^i}{E}\left[\frac{P_t}{P_{t+1}\ {}^{i}C_t^{t+1}}\right]\right)\frac{\partial\, {}^{i}C_t^t}{\partial L_t^i}\right] = 0.$$

Thus, (15) determines L_t^i as dependent only on expectations; substituting into ${}^{i}C_t^t$ yields consumption demand as a function of P_t, A_t and expectations.

Since all agents are identical and the system is stationary, $L^a = L_{t+1}^a = L_t^a = N\cdot L_t^i$; further, if all agents behave optimally, then <u>ex post</u> actual rebates will be:

$$(17)\quad R_{t+1}^i = \tau\cdot P_{t+1}A_{t+1}(L^a/N)$$

Define:

$$(18)\quad y_t^i = [{}^{i}C_t^t / A_tL_t^i];\ y_t^a = [N\,{}^{i}C_t^t / A_tL_t^a];$$

these are the individual and aggregate average propensities to consume out of gross income. From (9):

$$(19)\quad (\bar{M}/P_t) = A_tL_t^a\ (1-\tau-y_t^a).$$

Substituting (14) - (18) in (13) yields:

$$(20)\quad \frac{1}{y_t^i} = \lambda\cdot(P_tA_tL_t^i)\ E\left(\left[P_tA_tL_t^i((1-\tau)-y_t^i)+\tau P_{t+1}A_{t+1}(L^a/N)\right]^{-1}\right)$$

Since all agents are identical, $y_t^a = y_t^i = y_t$, $L_t^i = L^a/N$ and:

$$(21)\quad \left(\frac{1-\tau-y_t^a}{y_t^a}\right) = \lambda \cdot \underset{A_{t+1}}{E} \left[\frac{1-\tau-y_{t+1}^a}{1-y_{t+1}^a}\right]$$

Note that (21) must hold for all A_t; hence y_t^a is independent of A_t; further, the functions y_t^a and y_{t+1}^a must be identical. Therefore, the solutions to (21) are:

$$(22)\quad y_1^a \equiv (1-\tau) \text{ or } y_2^a = \left(\frac{1}{1+\lambda}\right)$$

Before discussing the cause of the multiple equilibrium, let us solve for L_t^i. For plan (1):

$$(23)\quad R_{t+1}^i = \tau \cdot P_{t+1} A_{t+1} \cdot L_t^i \left(\frac{L_{t+1}^a}{L_t^a}\right); \quad \frac{\partial R_{t+1}^i}{\partial L_t^i} = \tau P_{t+1} A_{t+1} = \frac{\tau \bar{M}}{L^a(1-\tau-y)}$$

For plan (2):

$$(24)\quad R_{t+1}^i = \tau\, P_{t+1}\, A_{t+1}\, (L_{t+1}^a/N); \quad \frac{\partial R_{t+1}^i}{\partial L_t^i} = 0.$$

Let L_j be individual labor supply under Social Security plan j; substitution into (15) yields:

$$(25)\quad L_1 = [\lambda/(1-y+\lambda)];$$

$$(26)\quad L_2 = \lambda(1-\tau)/[1-y+\lambda(1-\tau)]; \ \partial L_2/\partial\tau < 0, \ y < 1.$$

Finally, utility is determined from (12'), given:

$$(27)\quad {}^iC_t^{t+1} = A_{t+1}(L_{t+1}^a/N)\ [1-y_{t+1}] = A_{t+1}(L^a/N)(1-y).$$

Thus:

$$(28)\quad U_t = \ln\,[1-L_j] + (1+\lambda)\ \ln\,[L_j] + \ln(y) + \lambda\,\ln(1-y) + \ln A_t + \lambda\,\ln A_{t+1}.$$

Now that all the calculations are performed, some discussion is in order. The first point to note is that there are two consumption rules that are consistent with rational expectations; this is a familiar case of self-fulfilling expectations. Suppose that individuals at t believe all individuals at (t+1) will do no private savings ($y_{t+1} = (1-\tau)$); in this case it is clear their money balances will be worthless, and hence they have no incentive to save ($y_t=(1-\tau)$). This belief, then, that money has no value is a self-fulfilling one; and note that, while it is consistent with rational expectations, it is inefficient (unless τ happens to be $\lambda/1+\lambda$)).

Let us concentrate, then, on the alternative solution $y^*=[1/(1+\lambda)]$, clearly, the consumption rule is independent of τ. Further, for plan (1), L_1 is also independent of τ; hence, the proportional Social Security plan has no real effect in this case (as in the case of certainty), its only effect being nominal - to increase prices as τ increases.

On the other hand, for the flat rebate plan (2), labor effort decreases as τ increases, so that - despite rational expectations - the plan does affect real variables. Further, as can readily be shown from (29), any $\tau\neq 0$ reduces utility (realized or expected); hence, for this utility function, a flat rebate plan can only lower utility - again analogous to the case of certainty.

To sum up, for the given preferences, a proportional tax scheme has no real effect, whereas a flat one distorts the labor supply decision and lowers expected utility. However, these results only hold for $\rho=0$, because in the logarithmic case, private decisions are socially optimal. In part B of this section we show that the socially optimal consumption rule is $y^*=(1+\lambda^{1/1-\rho})^{-1}$ for all A_t, and the socially optimal labor supply is given by: $L^*=[1+(1+\lambda^{1/1-\rho})^{-1}(E(A^\rho))^{\frac{1}{\rho-1}}]^{-1}$. For the logarithmic case ($\rho=0$), these are the rules that private agents obey. Thus, if A_t is X% above (below) average, the working generation will increase (decrease) its consumption by X%. The retired generation will find that their savings yield X% more (less) consumption than on average. Thus, both generations share equally in the deviation of output from average output, and there is no room for policy to yield positive social benefits. When $\rho\neq 0$, private and socially optimal consumption and labor supply rules differ. For these cases, a Social Security plan--even the flat rebate plan that distorts labor incentives--can raise individual expected utility.

B) The General Case of Additive, Constant Relative Risk Aversion Preferences

Before proceeding with the equilibrium analysis, let us first inquire into what an efficient allocation of resources would be. As always under uncertainty, the notion of efficiency must be defined; specifically, then, we are looking for a stationary labor supply and consumption rule that will maximize the expected utility of a representative individual. Preferences are given by:

(29) $U_t = (1/\rho)[(1-L_t^i)^\rho + ({}^iC_t^t)^\rho + \lambda({}^iC_t^{t+1})^\rho], \ \rho < 1, \ \rho \neq 0.$

As eariler, let:

(30) $y_t = (C_t^t / A_t L_t);$

in this context, y_t is the fraction of output at t that will be consumed by the new generation; hence:

(31) $C_t^{t+1} = A_{t+1} L_{t+1} - C_{t+1}^{t+1} = A_{t+1} L_{t+1} (1-y_{t+1}).$

We seek a labor supply decision, L, and a consumption rule, $y(A_i)$, that maximized expected utility:

(32) $U_t = (1/\rho)\left[(1-L)^\rho + \underset{A_t}{E}\cdot(A_t^\rho L^\rho [y(A_t)]^\rho) + \lambda \underset{A_{t+1}}{E}(A_{t+1}^\rho L^\rho [1-y(A_{t+1})]^\rho)\right]$

Since A_t and A_{t+1} are identically distributed, (32) becomes:

(33) $U_t = (1/\rho)\left[(1-L)^\rho + L^\rho \underset{A}{E}[A^\rho\left(y(A)^\rho + \lambda(1-y(A))^\rho\right)]\right]$

Note that we must choose y for each A, i.e., we seek the function y(A) that maximizes (33). This, in essence, is an optimal control problem, and it is clear that the solution is given by:

(34) $y^* = \dfrac{1}{\left(1+\lambda^{\frac{1}{1-\rho}}\right)}$ for all A.

Optimizing (33) over L yields:

(35) $L^* = 1/[1 + (1 + \lambda^{\frac{1}{1-\rho}})^{-1} (E(A^\rho))^{\frac{1}{\rho-1}}]$

Note that for $\rho = o$, $y^* = (\frac{1}{1+\lambda})$, $L^* = (\frac{1+\lambda}{2+\lambda})$, which is one of the rational expectation solutions for the logarithmic case. Thus, (34) and (35) yield an efficient allocation pattern[4], and private decisions are socially efficient for the case $\rho = 0$.

Let us now consider the rational expectations equilibrium for $\rho \neq 0$. Given P_t, A_t, L_t^i, the individual chooses ${}^iC_t^t$ to maximize expected utility using (14):

$$(14') \quad {}^iC_t^{t+1} = [P_t(A_t L_t^i(1-\tau) - {}^iC_t^t) + R_{t+1}^i]/P_{t+1} \,,$$

we find:

$$(36) \quad ({}^iC_t^t)^{\rho-1} = \lambda E\left[\left(\frac{P_t}{P_{t+1}}\right)\left({}^iC_t^{t+1}\right)^{\rho-1}\right]$$

defines ${}^i\overline{C}_t^t\,(A_t, P_t, L_t^i, \tau)$.

Substituting in (29):

$$(37) \quad U_t^i = (1/\rho)\left[(1-L_t^i)^\rho + \underset{A_t, P_t}{E}\left(({}^i\overline{C}_t^t)^\rho + \lambda \underset{P_{t+1}}{} \underset{R_{t+1}^i}{E} ({}^iC_t^{t+1})^\rho\right)\right]$$

Optimizing over L_t^i yields:

$$(38) \quad (1-L_t^i)^{\rho-1} = \lambda\, E\left[\left(\frac{P_t A_t(1-\tau) + \dfrac{\partial R_{t+1}^i}{\partial L_t^i}}{P_{t+1}}\right)\left({}^iC_t^{t+1}\right)^{\rho-1}\right]$$

Equations (36) and (38) determine ${}^iC_t^t$, L_t^i, given expectations; assuming taxes are a constant rate, expectations are consistent and all individuals are alike, (36) can be rewritten as:

$$(39) \quad (A_t L_t^i)^{\rho-1}\,(y_t^i)^{\rho-1} = \lambda \underset{A_{t+1}}{E}\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{P_t A_t L_t^i(1-\tau-y_t^i) + \tau\, P_{t+1}\, A_{t+1}(L^a/N)}{P_{t+1}}\right)^{\rho-1}\right]$$

Using (9) - the price determination equation - and the identity of agents - (39) reduces to:

$$(40) \quad A_t^\rho\, y_t^{\rho-1}(1-\tau-y_t) = \lambda \cdot \underset{A_{t+1}}{E}\,[A_{t+1}^\rho(1-\tau-y_{t+1})(1-y_{t+1})^{\rho-1}]$$

Since the LHS (left hand side) of (40) must hold for all A_t, it follows that y_t depends only on A_t and τ; and is independent of L_t:

$$(41) \quad \frac{\partial y_t}{\partial A_t} \gtreqless 0 \text{ as } \rho \gtreqless 0.$$

Thus, for $\rho=0$, this reduces to the same solution as the previous section.

For $\rho\neq 0$, two solutions exist:

(42) $y_1(A)\equiv(1-\tau)$, or

(42') $y_2(A);\ \frac{\partial y_2}{\partial A}\neq 0,\ \rho\neq 0.$

Assuming $y_2(A)\neq(1-\tau)$, theoretically (40) can be solved for $y(A)$, given the distribution of A (the functions $y_t(A_t)$ and $y_{t+1}(A_{t+1})$ are identical, given identical preferences and identical distributions for A_t, A_{y+1}). In general, this solution will depend on the distribution of A, and hence we cannot present any precise analytical solution.

Given the solution, $y(A)$, the individual labor supply decision for each plan is determined from (38); after some simplification we find; for plan (1):

$$(43)\quad \left(\frac{1-L_1}{L_1}\right)^{\rho-1} = \underset{A_t}{E}\left[A_t^{\rho}\ y(A_t)^{\rho}\right] + \lambda\ E\left[(A_{t+1})^{\rho}\ (1-y(A_{t+1}))^{\rho}\right]$$

$$= (1-\tau)\ \underset{A_t}{E}\left[A_t^{\rho}(y(A_t))^{\rho-1}\right] + \lambda\tau \underset{A_{t+1}}{E}\left[A_{t+1}^{\rho}(1-y(A_{t+1}))^{\rho-1}\right]$$

which, given the distributions, could (theoretically) be solved for L_1. For plan (2), $\frac{\partial R_{t+1}^{i}}{\partial L_t^{i}}=0$, and (38) reduces to:

$$(44)\quad \left(\frac{1-L_2}{L_2}\right)^{\rho-1} = (1-\tau)\ \underset{A_t}{E}\left[A_t^{\rho}(y(A_t))^{\rho-1}\right]$$

Again, given the distribution of A, this could (theoretically) be solved for L_2.

Thus, (40) and (43) or (44) determine individual - and hence aggregate - behavior. From (40), as noted earlier, two solutions are possible. The first, $y\equiv(1-\tau)$, is the self-fulfilling rational expectations solution that renders money worthless Note that, for arbitrary τ, this solution is inefficient, a not uncommon characteristic of the self-fulfilling rational expectations solution.

For $y<(1-\tau)$, it can be shown that - given τ - a unique solution (i.e., a unique function y(A) that solves (40)) exists[5]; note that for $\rho \neq 0$, this solution will not be efficient in the sense defined earlier. Thus, it is feasible for a Social Security plan to increase welfare.

Using the fact A_t and A_{t+1} are identically distributed, (40) can be rewritten as:

$$(40') \quad \lambda \underset{A}{E}\left[\left(\frac{1-y(A)}{y(A)}\right)^{\rho-1}\right] = 1, \quad y < (1-\tau)$$

Hence:

$$(45) \quad \left(\frac{1-y}{y}\right) > \left(\frac{\tau}{1-\tau}\right) \text{ and } 1 = \lambda \underset{A}{E}\left[\left(\frac{1-y}{y}\right)^{\rho-1}\right] < \lambda \left(\frac{1-\tau}{\tau}\right)^{(1-\rho)}$$

For $\tau > \tau^* = \left[\lambda^{\frac{1}{1-\rho}} / (1+\lambda^{\frac{1}{1-\rho}})\right]$, no solution to (40') exists; hence, for $\tau > \tau^*$, $y = (1-\tau)$ is the only solution. Furthermore, as $\tau \to \tau^*$ from below, $y \to (1-\tau^*)$; at τ^*:

$$(46) \quad y(\tau^*) = 1-\tau^* = \left[1/\left(1+\lambda^{\frac{1}{1-\rho}}\right)\right],$$

the effecient consumption rule found earlier. Further, at τ^*, from (43), using plan (1), labor is given by:

$$(47) \quad L_1(\tau^*) = 1/\left[\left(1+\lambda^{\frac{1}{1-\rho}}\right)^{-1} \left(E(A^\rho)\right)^{\frac{1}{\rho-1}} + 1\right]$$

Thus, for the proportional plan, the efficient allocation can be achieved by a tax rate of (τ^*); in the absence of this plan, private decisions would be inefficient.[6]

In addition, one could readily demonstrate the impact of changes in the tax rate on expected utility and labor supply. From (37):

$$(48) \quad E[U_t] = (1/\rho)\ [(1-L)^\rho + \underset{A_t}{E}\ (A_t^\rho \cdot L^\rho \cdot y^\rho) + \lambda \underset{A_{t+1}}{E}\ (A_{t+1}^\rho\ L^\rho\ (1-y)^\rho)]$$

$$= (1/\rho)[(1-L)^\rho + L^\rho \underset{A}{E}\ (A^\rho[y^\rho + \lambda(1-y)^\rho])\ .$$

In (48), we employ the assumption all generations behave identically and that $(A\ ,\ A_{t+1})$ are identically distributed. Differentiating (48) with respect to the tax rate τ:

$$(49)\quad \frac{\partial E[U_t]}{\partial \tau} = L^{\rho-1}\left[E\left(A^{\rho}(y^{\rho}+\lambda(1-y)^{\rho}\right) - \left(\frac{1-L}{L}\right)^{\rho-1}\right]\frac{\partial L}{\partial \tau}$$

$$+ L^{\rho}\cdot E\left[A^{\rho}\left(y^{\rho-1}-\lambda(1-y)^{\rho-1}\right)\frac{\partial y}{\partial \tau}\right]$$

For the proportional plan, the first expression in brackets on the RHS is zero, as can be seen from (43). In addition, it can be demonstrated that the second expression on the RHS is positive for $\tau < \tau^*$; hence, expected utility is monotonically increasing in τ - up to the optimal tax rate - for this plan.[7] Also, from (43) we can determine how the labor supply is affected by the tax rate:

$$(50)\quad (1-\rho)\,\frac{(1-L_1)^{\rho-2}}{L_1^{\rho}}\,\frac{\partial L_1}{\partial \tau} = \rho\cdot E\left[A^{\rho}\left(y^{\rho-1}-\lambda(1-y)^{\rho-1}\right)\frac{\partial y}{\partial \tau}\right] \gtreqless 0 \text{ as } \rho \gtreqless 0,\ \text{for } \tau < \tau^*.$$

Thus, increases in the tax rate can either raise or lower the labor supply depending on the degree of risk aversion. Note that there is no presumption that the actual labor supply will be decreased as a result of the tax-transfer plan.

Consider next the second tax scheme - the one in which the next period transfer is independent of current effort. From (43) and (44) it is immediately apparent that, for $\tau > 0$, less labor will be supplied under plan 2 than plan 1: $L_2 < L_1$ for $\tau > 0$. Intuitively, this is quite reasonable since this second scheme lowers the (expected) return to labor, therefore causing individuals to substitute leisure for consumption (and labor).

The impact of this tax plan on expected utility can be found from substituting (44) into (49) and simplifying:

$$(51)\quad \frac{\partial E[U_t]}{\partial \tau} = (L_2)^{\rho-1}\left(\frac{\partial L_2}{\partial \tau}\right)\left[\lambda\tau E(A^{\rho}(1-y)^{\rho-1})\right] + (L_2)^{\rho}\ E\left[\left(A^{\rho}y^{\rho-1}-\lambda(1-y)^{\rho-1}\right)\frac{\partial y}{\partial \tau}\right]$$

where, from (50)

$$(52)\quad (1-\rho)\ \frac{(1-L_2)^{\rho-2}}{L_2^{\rho}}\ \frac{\partial L_2}{\partial \tau} = -E\left[A^{\rho}y^{\rho-1}\right] - (1-\tau)(1-\rho)\ E\left[A^{\rho}y^{\rho-2}.\frac{\partial y}{\partial \tau}\right]$$

Consider (51) first; since the second term on the RHS in (51) is positive, it immediately follows that, for $\tau=0$, a small increase in the tax rate will increase expected utility. Thus, for small tax rates, the benefits of intergenerational risk-sharing dominate the costs imposed by the labor distortion introduced by the plan.

However, as the tax rate increases, the net impact of further increases in it on expected utility are ambiguous; on the one hand, it is beneficial because of the risk-sharing, but on the other hand, the decrease in the labor supply induced by higher taxes lowers expected utility.[8] Consequently, for this second best plan, there will be an optimal tax rate $\hat{\tau} < \tau^*$; the magnitude of this tax rate will, in general, depend upon ρ. As it is impossible to provide an analytic solution for this "second-best" tax rate, in the next section we shall provide some simulation experiments that estimate its value.

To sum up the results of this section, we have found for the proportional tax plan that:

(i) there exists an optimal tax rate that supports the efficient allocation;

(ii) individual expected utility is increasing in the tax rate, for tax rates less than the optimal rate;

(iii) the effect on labor supply depends on the degree of risk aversion; higher tax rates - under this plan - can either raise or lower actual labor supply. In contrast, for the flat tax plan, in which transfers are independent of that individual's labor supply, we have seen that:

(iv) labor supply is always lower under this plan than under the proportional plan ($\tau>0$);

(v) due to substitution effects - the net impact of the tax is to lower labor supply;

(vi) the effect of increases in the tax rate on expected utility is ambiguous, due to offsetting effects of labor distortions and risk-sharing. However, for τ near zero, increases in τ increase expected utility, whereas for τ near τ^* they decrease it. Consequently, a positive optimal tax rate exists for this plan. In the next section we provide simulation results that calculate this optimal tax rate, and show how its value depends on ρ and the variance of A.

III. Simulation Results

In the previous section, it was shown that for the proportional tax plan, increases in the tax rate, up to the optimal rate, increase (decrease) the supply of labor if ρ is positive (negative). Further, the optimal tax rate is given by:

$$(53)\quad \tau^* = \left[\lambda^{1/1-\rho}/(1+\lambda^{1/1-\rho})\right], \ \rho \neq 0.$$

However, for the flat rebate plan it was not possible to obtain analytic solutions for the second best tax rate or to demonstrate whether labor supply and expected utility are monotonic for a less-than-optimum tax rate. In this section we simulate[9] the model in order to provide the magnitudes of changes under the proportional plan and to find the optimum tax rate in the distortionary scheme. Furthermore, we also discuss the extent to which Social Security taxation alters private saving.

A) Consumption and Labor Supply Effects

We simulate the model using the general form of the utility function given by equation (12) for several different assumptions concerning the distribution of the output disturbance (A_t). In all cases it is assumed that the output disturbance is independently distributed with mean equal to unity. Our first set of simulations assumes that the output disturbance is drawn from a symmetric binomial distribution. For the three cases we considered, the output disturbance could have realized values of .8 or 1.2 so that Var(A) = .04; .5 or 1.5 so that Var(A) = .25; or .2 or 1.8 so that Var (A) = .64. For this set of simulations, tax rates between 0.0 and .5 were considered at intervals no greater than .01. We examined all ρ in the range 0.9 to -.9 at intervals of .1 for values of λ = 1.0 and 0.8. Fortunately, the endogenous variables in the system (including expected utility) are monotonic in ρ, λ, and Var (A) so that only selected results need be presented. Another set of simulations considered the case in which the output disturbance was drawn from a log-normal distribution. As the qualitative results appear to be robust to the form of the distribution, we present a single example of the case in which the output disturbance is log-normally distributed.

Table 1 presents the optimal tax rates for alternative values of ρ and the rate of time preference. Notice that the optimal tax rates for the proportional plan are distribution-free and are monotonic in rho if $\lambda < 1.0$. Also, note that--as demonstrated by equation (50)--there is a positive tax rate that maximizes expected utility for the distortionary Social Security scheme. In Table 1 the optimal tax rates for plan 2 are such that each satisfies the maximization problem in equations (51) and (52). Further, as discussed above, the optimal tax rate τ^* is always greater than the tax rate that maximizes expected utility under the distortionary scheme ($\hat{\tau}$). Probably the most important demonstration in Table 1 is that the "second-best" optimal tax rate is an increasing function of the variance of the output disturbance. This is easily

TABLE 1

OPTIMAL TAX RATES
(BINOMIAL DISTRIBUTION)

	RHO							
	0.8	0.6	0.4	0.2	-0.2	-0.4	-0.6	-0.8
Proportional tax (All distributions)								
$\lambda = 1.0$	.5	.5	.5	.5	.5	.5	.5	.5
$\lambda = .8$	.2468	.3640	.4081	.4307	.4536	.4602	.4652	.4690
Flat rebate								
Var (A) = .04[1]								
$\lambda = 1.0$	*	*	*	*	*	<.01	.01	.02
$\lambda = .8$	*	*	*	*	*	<.01	.01	.03
Var (A) = .25[2]								
$\lambda = 1.0$	.020	.030	.020	.010	.010	.040	.080	.125
$\lambda = .8$	.015	.030	.025	.010	.010	.045	.090	.145
Var (A) = .64[1]								
$\lambda = 1.0$	.04	.07	.06	<.01	.04	.12	.23	.49+
$\lambda = .8$	.04	.07	.06	<.01	.04	.14	.27	.46

Note: * Not detectable on the intervals used.

1) Tax rate to nearest hundredth
2) Tax rate to nearest five thousandth.

understood, once it is remembered that a Social Security plan is a means by which intergenerational risk sharing can be increased. The greater the risk (i.e., the greater Var (A)) the greater are the marginal benefits to be derived from risk sharing. Thus, the Barro-Samuelson contention concerning the irrelevance of fully-funded Social Security programs cannot be extended to an uncertain economic environment: even distortionary programs can increase social welfare. While the second best tax rate in the flat rebate plan is not monotonic in rho or the discount rate, a high degree of relative risk aversion is associated with a relatively large "optimal" Social Security tax rate

Tables 2 and 3 indicate the effects of Social Security taxes on the supply of labor and on the average propensity to consume. Space considerations prohibit consideration of all values of ρ and the tax rate, but the results presented are quite representative of all our results. Before we examine specific cases, some general remarks are in order. For the proportional tax scheme, the magnitude of the change in the supply of labor is quite small. Yet, as indicated in Section II:

1) The supply of labor under the proportional tax scheme is always greater than that of the distortionary scheme for corresponding tax rates not equal to zero.
2) Under the flat rebate plan, increases in the tax rate strongly discourage work. For the proportional plan, increasing the tax rate may act to increase or decrease the supply of labor depending upon the degree of relative risk aversion and whether $\tau \gtrless \tau^*$. The supply of labor is maximized (minimized) at a tax rate of τ^* if ρ is positive (negative).
3) Under the proportional plan, the socially optimal labor supply (L^*) and average propensity to consume (y^*) occur at τ^*. For the flat rebate plan, in some cases it is possible to obtain either the socially optimal labor

TABLE 2

SUPPLY OF LABOR

RHO	TAX	PROPORTIONAL PLAN VAR (A)=.04	PROPORTIONAL PLAN VAR (A)=.64	FLAT REBATE TAX VAR (A)=.04	FLAT REBATE TAX VAR (A)=.64
		No time preference (λ=1.0)			
.8	0.0	.66145	.55252	.66145	.55252
	.1	.66196	.56557	.53547	.41626
	.2	.66240	.57772	.39014	.27274
	.3	.66275	.58522	.24728	.14905
	.4	.66298	.59055	.13223	.09401
	$\tau^*=$ .5	.66306	.59241	.06358	.04775
.2	0.0	.665749	.64427	.665749	.64427
	.1	.665752	.64434	.635812	.61295
	.2	665754	64442	.601723	.57700
	.3	.665757	.64450	.560450	.53558
	.4	.665759	.64457	.512582	.48777
	$\tau^*=$ .5	.665760	.64460	.461922	.43873
-.8	0.0	.670578	.75628	.670578	.75628
	.1	.670533	.75588	.657320	.74252
	.2	.670480	.75533	.642205	.72618
	.3	.670413	.75454	.624758	.70677
	.4	.670339	.75337	.604362	.68585
	$\tau^*=$ .5	.670289	.75245	.583076	.67649
		Positive time preference (λ=.8)			
.8	0.0	.56568	.47409	.56568	.47409
	.1	.56614	.48507	.43473	.34759
	.2	.56639	.49046	.29946	.23510
	$\tau^*\sim$ .25	.56642	49105	.23667	.18695
	.3	.56638	49031	.18035	.14422
-.8	0.0	65719	.74514	.65719	.74514
	.1	.65713	.74466	.64365	.73069
	.2	.65707	.74398	.62826	.71355
	.3	.65699	.74299	.61055	.69352
	.4	.65691	.74165	.58998	.67424
	$\tau^*\sim$ .47	.656882	.74110	.57365	.66816
	.48	.656883	.74112	.57119	.66802

TABLE 3

AVERAGE PROPENSITIES TO CONSUME

RHO	TAX	VAR (A)=.04 A_t=.8	A_t=1.2	VAR (A)=.64 A_t=.2	A_t=1.8
		No time preference (λ=1.0)			
.5					
	0.0	.4651	.5326	.2920	.6583
	.1	.4702	.5281	.3233	.6314
	.2	.4761	.5228	.3604	.6095
	.3	.4828	.5166	.4032	.5812
	.4	.4907	.5091	.4504	.5451
	τ^* = .5	.5000	.5000	.5000	.5000
-.5					
	0.0	.5190	.4785	.5748	3621
	.1	.5174	.4806	.5710	.3749
	.2	.5152	.4832	.5651	.3923
	.3	.5122	.4868	.5552	.4171
	.4	.5076	.4920	.5367	4529
	τ^* = .5	.5000	.5000	.5000	.5000
		Positive time preference (λ=0.8)			
.5					
	0.0	.5784	.6383	.4088	.7357
	.1	.5851	.6326	.4564	.7149
	.2	.5927	.6260	.5088	.6872
	.3	.6011	.6182	.5629	.6508
$.39<\tau^*<.4$	.39	.60973	.60978	.6096	.6098
	.4	.6107	.6088	.6146	.6048
-.5					
	0.0	.5562	.5154	.6113	.3948
	.1	.5543	.5178	.6069	.4106
	.2	.5517	.5210	.5998	.4324
	.3	.5480	.5253	.5873	.4633
	.4	.5424	.5316	.5632	.5059
$.46<\tau^*<.47$	.46	.5374	.5368	.5385	.5356
	.47	.5367	.5378	.5335	.5406

Note: The average propensities to consume are invariant to the tax scheme. Thus, the figures above are the average propensity to consume under either plan.

supply or average propensity to consume (but not both).

4) It is of interest to note that under either tax scheme, the smaller is rho, the greater is the supply of labor. Thus, the results do not bear out the claim that risk aversion would induce individuals to substitute the certainty of leisure for the uncertainty of market activities. Rather as risk aversion increases, individuals are drawn into work activities in order to provide for consumption in retirement.[10]

Notice that the direction of change of the labor supply and average propensity to consume depend upon the sign of ρ. Consider first the case in which rho is positive. Clearly--as indicated by equation (41)--$\partial y/\partial A_t$ is positive. For the case of a binomial distribution, the effects of an increase in the tax rate is to increase y for $y<y^*$ and to decrease y for $y>y^*$. At τ^*, the two values of y are equal. Thus, for $\tau<\tau^*$, low-output generations will consume a smaller proportion of their output than is socially optimal and high-output generations will consume a larger proportion than is socially optimal. To see why this is important, consider the special case in which $\lambda=1$ so that $y^*=.5$. When the actual value of y is .5, a generation that has an output, say 50%, lower (higher) than average will reduce (increase) its consumption by 50%. The retired generation will find that real cash balances purchase 50% fewer (more) goods than average. Thus, both generations share equally in the output uncertainty. In the case under consideration, however, low-output generations will reduce their consumption by a relatively small amount (as $y<.5$) and high-output generations will increase their consumption by a relatively large amount (as $y_\tau>.5$). As such, there tends to be less intergenerational risk sharing. Increases in the tax rate, for $\tau<\tau^*$ act to spread the uncertainty across generations. In doing so, as can be seen from Table 2, individuals will be drawn into work activities under the proportional tax scheme. With the flat rebate plan, the distortionary effects are to reduce the supply of labor.

Thus, the flat rebate plan reduces consumption uncertainty at the expense of distorting the labor supply decision.

Again considering positive values for ρ, the larger is the variance of the output disturbance, the smaller is the supply of labor. High-output variance also acts to decrease the average propensity to consume of low-output generations and increase the average propensity to consume of high-output generations. Thus, output uncertainty acts to move the economy away from the optimal consumption and labor supply rules.

Considering the case in which people are very risk averse ($\rho<0$), high tax rates act to decrease the supply of labor and to decrease (increase) the average propensity to consume of low (high) output generations. High output variance acts to increase the supply of labor and to move the average propensity to consume away from y^*. In this case, the supply of labor will be less than L^* for all $\tau < \tau^*$; and $y > y^*$ for low-output generations, but $y < y^*$ for high-output generations.

Combined with Section II, the simulation results indicate that Social Security can pool risks intergenerationally. For risk averse individuals, then, social welfare can be increased by resorting to an intergenerational transfer of commodities via Social Security taxation. These results appear to be quite robust to the distribution of the output disturbances. In Table 4, we present the effects of the two Social Security plans on labor supply and expected utility when disturbances are log-normally distributed. The direction of change in expected utility and work effort are in complete accord with the results for the binomial distribution. When rho is positive (and $\lambda = 1$), an increase in the tax rate acts to increase labor supply under the proportional plan and reduce it with a flat rebate. Expected utility is largest under the proportional plan with a tax rate of .5. Note

TABLE 4

LOG-NORMAL DISTRIBUTION

Tax rate	Proportional plan: Work effort (Labor supply)	Proportional plan: Expected utility	Flat rebate plan: Work effort (Labor supply)	Flat rebate plan: Expected utility
0.00	.654505	3.402584	.654505	3.402584
$\hat{\tau}$ = .02	.654576	3.402936	.645168	3.402770
.04	.654647	3.403287	.635535	3.402608
.10	.654859	3.404332	.604773	3.399749
.20	.655198	3.406005	.546903	3.385107
.30	.655501	3.407410	.480311	3.353771
.40	.655731	3.408639	.405108	3.299413
.48	.655823	3.409097	.339605	3.234159
τ^* = .50	.655828	3.409119	.322668	3.214253

Notes: 1) We have assumed that $\rho = .5$ and $\lambda = 1.0$. Thus, $\tau^* = .5$.

2) The distribution of A is such that $E(A) \sim 1.0$ and VAR $(A) \sim .475$. The log-normal distribution was approximated through twenty-five possible realizations of A_t. Let B_i $(i = 1, \ldots 25)$ be such that the B_i divide the log-normal frequency distribution into the twenty-five equal areas a_i $(i = 1, \ldots 25)$. The A_i are drawn such that they divide each a_i into two equal areas.

that under the flat rebate scheme, there is a positive second-best tax rate.

B) Saving Effects

We now consider the effects of Social Security taxation on saving decisions. While money is the only asset in the model, and the nominal money supply is fixed because we only consider fully funded Social Security programs, the private sector can alter saving via changes in the price level. Recall from equation (9) that the real value of cash balances is given by:

$$(9) \quad \frac{\overline{M}}{P_t} = A_t L_t^a (1-\tau) - \overline{C}_t^a.$$

As real cash balances can be considered to be total private saving (net of Social Security), equation (9) indicates that actual or realized saving will be dependent upon the realization of A_t. Thus, it is meaningful to consider realized private savings ($\overline{M}/P_t$ or RPS) and expected private savings ($E[\overline{M}/P_t]$ or EPS). Additionally, it is possible to consider the amount that the retired generation consumes (M/P_t + the rebate) and the amount the retired generation can be expected to consume (i.e., expected private saving plus the expected rebate), or expected total saving (ETS). Thus, expected total saving is given by:

$$(54) \quad ETS = E[\overline{M}/P_t + \tau A_t L_t^a] = E[A_t L_t^a - \overline{C}_t^a].$$

Table 5 indicates the extent to which the various concepts of saving are affected by the tax scheme, tax rate, rho and the discount rate (λ). All simulations are for the case in whicn A_t is drawn from a bionomial distribution. The results presented in the table are representative of all rho between .9 and -.9. Clearly, for all cases, high output realizations lead to high realized private saving, and Social Security taxation does <u>not</u> lead to a dollar-for-dollar reduction in private saving. Rather, the flat rebate plan always acts to reduce both expected private saving and expected total saving.

TABLE 5

SAVING BEHAVIOR

RHO	TAX	Proportional plan: RPS Low Realization	Proportional plan: RPS High Realization	Proportional plan: EPS	Proportional plan: ETS	Flat rebate: RPS Low Realization	Flat rebate: RPS High Realization	Flat rebate: EPS	Flat rebate: ETS
				No time preference (λ=1.0) VAR (A) = .04					
.5	0.0	.2842	.3725	3283	.3283	.2842	.3725	.3283	.3283
	.1	.2284	.2964	.2624	.3288	.2116	.2747	.2432	.3047
	.2	.1721	.2209	.1965	3294	.1447	.1857	1652	.2769
	.3	.1154	.1462	.1308	.3301	.0855	.1083	.0969	.2444
	.4	.0581	.0725	.0653	.3310	.0364	.0453	.0408	.2072
	.5	.0000	.0000	.0000	.3322	.0000	.0000	.0000	.1700
-.5	0.0	.2574	.4187	.3380	.3380	.2574	.4187	.3380	.3380
	.1	.2048	.3367	.2707	.3376	.1999	.3288	.2643	3297
	.2	.1524	.2543	.2034	.3371	.1447	.2414	.1931	.3201
	.3	.1005	.1711	1358	.3365	0923	.1571	.1247	.3089
	.4	.0494	.0867	.0681	.3356	.0436	.0764	.0600	.2958
	.5	.0000	.0000	.0000	.3345	.0000	.0000	.0000	.2816
				VAR(A) = .64					
.5	0.0	.0862	.3852	.2357	.2357	.0862	.3852	.2357	.2357
	.1	.0704	.2951	.1827	.2438	.0639	.2677	1658	.2211
	.2	.0538	.2099	.1318	.2542	.0433	.1689	.1061	.2047
	.3	.0364	.1312	.0838	.2679	0253	.0912	.0583	.1862
	.4	.0184	.0607	.0396	.2855	.0107	.0353	.0230	.1657
	.5	.0000	.0000	.0000	.3077	.0000	.0000	0000	.1469
-.5	0.0	.0616	.8322	.4469	.4469	.0616	.8322	.4469	.4469
	.1	.0477	.6848	.3663	4387	.0466	.6698	.3582	.4291
	.2	.0340	.5314	.2827	.4276	.0324	.5063	.2693	.4073
	.3	.0210	.3685	.1947	.4119	.0194	.3403	.1798	.3803
	.4	.0092	.1916	.1004	.3897	.0082	.1707	.0894	.3472
	.5	.0000	.0000	.0000	.3615	.0000	.0000	.0000	.3125

In the case of the proportional plan, Social Security taxation acts to reduce expected private saving, but may act to increase or decrease expected total saving. Thus, if rho is negative, there is --in an expected sense--more than a dollar-for-dollar reduction in private saving for either scheme. If rho is positive, less than a dollar-for-dollar reduction occurs for the proportional plan.

The effects of output variations on saving are not always monotonic. However, under either tax plan, if rho is positive, increasing the variance of the output disturbance acts to reduce both EPS and ETS for all tax rates less than τ^*. When rho is negative, both EPS and ETS increase as the variance of output increases. These results indicate that if individuals are strongly risk averse ($\rho<0$) they react to high output and price variability by saving for retirement. If individuals are less risk averse ($\rho>0$) they react to high output and price variability by consuming; thereby reducing the possibility of large capital gains or losses on their savings when retired. Lastly, it is possible to consider the effects of Social Security taxes on the realized consumption of the retired generation. While not shown directly in the tables, realized or actual consumption of the retired generation is negatively (positively) related to the tax rate in the proportional plan if $\rho(A_t-E[A])$ is negative (positive).

IV. Conclusions

We have shown that in an uncertain world, fully funded Social Security programs can increase social welfare and affect individual labor supply, consumption and saving decisions. This result was derived from a model in which individuals maximize expected utility and have rational expectations. Thus, there are economic reasons--as opposed to irrationality or myopia--which can be used to justify fully funded Social Security programs. Social Security can be used as a means to spread exogenous output and endogenous price uncertainty across generations.

The major limitation of our model is that we assume money is the only store of value. Allowing for storable commodities would certainly change many of our results. However, few commodities can be stored risk-free so that there are few ways to risklessly transfer consumption intertemporally. We also believe that it would be useful to consider Social Security programs in the context of a model that does allow for some--albeit risky--commodity storage.

FOOTNOTES

* The authors are Associate Professor and Professor of Economics at Iowa State University. We would like to thank Roy Gardner for his suggestions.

[1] We assume no bequest motive exists, so the new generation starts with no money; the money holdings of a member of the older generation equal the difference between that individual's net income and consumption expenditures from the previous period.

[2] Of course, since all individuals are alike, ex post the transfer to all individuals of the same generation will be identical. However, the two plans lead to different labor supply decisions since the individual, in choosing his labor-supply decision, (correctly) assumes his choice does not affect other agents. In other words, if individual 1 were to choose a sub-optimal labor decision, this would not induce others to do likewise. Consequently, under plan 1, an individual pays tax on each extra hour worked, but also receives an increase in expected benefits, whereas under plan 2 there is no offsetting increase in benefits due to an increase in L_t^i.

[3] Also, the general price level will tend to increase as the tax rate increases, since the demand for money will decrease. In particular, for tax rates less than 50%, prices must increase as τ increases.

[4] It should also be noted that this solution maximizes the expected value of a Bergsonian Social Welfare function that is additive in individual utilities.

[5] Details are omitted to conserve space; a proof is available from the authors upon request.

[6] If a private insurance system could be established whereby individuals pledged ex ante to contribute τ^* of the income in their work period in return for a comparable remuneration next period, then this plan would - of course - achieve an efficient allocation without government intervention. The problems that seem to arise for this scheme are: (i) how does the insurance company enforce its contract; people with an unusually good outcome for A_t may try to opt out (cancel) their plan. Hence, contracts must be made ex ante and must be enforceable; (ii) how does an individual know any one company will be solvent next period?; if 30% of the population engages in an insurance contract with company B, and only 25% join next year, the company cannot fulfill its contract. Hence, either there must be only one company, or agents must have perfect foresight (not just rational expectations) as to what insurance enrollments will be next period. While a private insurance plan might work in such an environment, it seems that it is logically equivalent to a government plan, and the government programs seem to reduce the uncertainty concerning enforcement of contracts and membership enrollments.

[7] The proof that $E[A^\rho(y^{\rho-1}-\lambda(1-y)^{\rho-1})\frac{\partial y}{\partial t}] > 0$, $\tau < \tau^*$, is rather tedious; since this demonstration would add little insight, it is omitted in order to conserve space. Interested readers may obtain a proof from the authors.

[8] For $\rho > 0$, labor effort monotonically decreases as the tax rate increases; for $\rho < 0$, we have not been able to show that labor effort monotonically declines with higher taxes. However, since labor effort under this

plan is less than that for the proportional plan, and since - for $\rho < 0$ - tax increases decrease labor supply under the proportional plan, the net impact of taxes under the flat rebate scheme is to reduce labor effort.

[9] The term "simulation" has come to have several different meanings, so that some explanation is in order. In order to solve for the endogenous variables in the model, it is necessary to specify values for rho, the rate of time preference, and the theoretical distribution of the output disturbance. Note that it is not necessary to specify a time path for actual realizations of the output disturbance: at the risk of being redundant, only the theoretical distribution need be specified.

[10] Note that for our utility function, ρ determines both the degree of risk aversion and the elasticity of substitution among "goods" Hence, our conclusions concerning how changes in ρ affect behavior reflect the impact of changes in both the degree of risk aversion and the degree of commodity substitutability.

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SUPPLEMENT TO "SOCIAL SECURITY TAXATION AND INTERGENERATIONAL RISK SHARING"
PROOF OF MONOTONICITY OF EXPECTED UTILITY IN TAX RATE

Let $\bar{U}$ = maximized expected utility. From 49 for positive L, sgn $[\partial\bar{U}/\partial\tau]$ is given by:

(1) $\frac{\partial\bar{U}}{\partial\tau} = E\left[A^{\rho}(y^{\rho-1}-\lambda(1-y)^{\rho-1})y_{\tau}\right]$, where $y_{\tau} = \frac{\partial y}{\partial\tau}$ (A). From (40):

(2) $A^{\rho}y^{\rho-1}(1-\tau-y) \equiv H(\tau) = \lambda\ E\left[A^{\rho}(1-\tau-y)(1-y)^{\rho-1}\right]$; from (40')

(3) $E\left\{\lambda\left[\left(\frac{1-y}{y}\right)^{\rho-1}\right]\right\} = 1$; hence:

(4) $E\left[\frac{(1-y)^{\rho-2}}{y^{\rho}} \cdot y_{\tau}\right] = 0$; therefore, y_{τ} changes signs.

From (2), since the LHS holds for all A:

(5) $y_{\tau} = \frac{y[\theta(1-\tau-y) - 1]}{(1-\rho)(1-\tau) + \rho y}$; $\theta \equiv -\left(\frac{dH/d\tau}{H}\right) > 0$

Thus, from (5), there exists:

(6) y^{*} s.t. $y_{\tau} \gtrless 0$ as $y \lessgtr y^{*}$; $y^{*} = \left[\frac{\theta(1-\tau)-1}{\theta}\right] > 0$.

Define:

(7) $\hat{y}$ s.t. $\lambda\left[\left(\frac{1-\hat{y}}{\hat{y}}\right)^{\rho-1}\right] = 1$; $\quad = \left[1\Big/\left(1+\lambda^{\frac{1}{1-\rho}}\right)\right]$

Rewrite (1):

(1') $\frac{\partial\bar{U}}{\partial\tau} = E\left[\frac{A^{\rho}y^{\rho-1}(1-\tau-y)}{(1-\tau-y)}\, y_{\tau}\left(1-\lambda\left(\frac{1-y}{y}\right)^{\rho-1}\right)\right] = H\cdot E\left[\left(\frac{y_{\tau}}{1-\tau-y}\right)\left(1-\lambda\left(\frac{1-y}{y}\right)^{\rho-1}\right)\right]$

From (4), for any ϕ, $\phi\ E\left[\left(\frac{1-y}{y}\right)^{\rho-2}\frac{y_{\tau}}{y2}\right] = 0$; insert in (1'):

(8) $\frac{\partial\bar{U}}{\partial\tau} = H\cdot E\left[\left(\frac{y_{y}}{1-\tau-y}\right)\left(1-\lambda\left(\frac{1-y}{y}\right)^{\rho-1}\right) + \phi\left(\frac{1-y}{y}\right)^{\rho-1}\left(\frac{1-\tau-y}{1-y}\right)\frac{1}{y}\right]$

$$= H\cdot E\left[\left(\frac{y_{\tau}}{1-\tau-y}\right)\left(\frac{1-y}{y}\right)^{\rho-1}\left(\frac{1}{y}\right)\left\{y\left[\left(\frac{1-y}{y}\right)^{1-\rho} - \lambda\right] + \phi\left(\frac{1-\tau-y}{1-y}\right)\right\}\right]$$

Define:

(9) $M(y) \equiv y\left[\left(\frac{1-y}{y}\right)^{1-\rho} - \lambda\right] + \phi\left(\frac{1-\tau-y}{1-y}\right)$

Choose ϕ s.t. $M(y^{*}) = 0$ (where $y_{\tau}(y^{*}) = 0$). Then:

(10) $\phi \gtrless 0$ as $y^* \gtrless \hat{y}$.

Note that

(11) $\lim_{y \to 0} M(y) \gtrless 0$ as $\phi \gtrless 0$ for $\rho > 0$

$\lim_{y \to 0} M(y) > 0$ for $\rho < 0$

(12) $M\,(1-\tau) < 0$ for $\tau < \tau^*$.

Also:

(13) $$\frac{dM}{dy} = \left(\frac{1-y}{y}\right)^{-\rho} - \lambda - (1-\rho)\, y^{\rho-1}\,(1-y)^{-\rho} - \frac{\phi\tau}{(1-y)^2} = (1-y)^{-\rho}\cdot y^{\rho-1}\,[\rho-y] - \lambda - \frac{\phi\tau}{(1-y)^2}$$

(14) $$\frac{d^2M}{dy^2} = -(1-\rho)\,\rho\cdot\left[(1-y)^{-\rho}\cdot y^{\rho-2} + y^{\rho-1}\,(1-y)^{-(\rho+1)}\right] - \frac{2\phi\tau}{(1-y)^2}$$

Consider $\rho < 0$. If $\phi > 0$, $\left({dM}/{dy}\right) < 0$ everywhere, and hence there is a unique $y(=y^*)$ s.t. $M(y^*) = 0$. If $\phi < 0$, $\frac{d^2M}{dy^2} > 0$; since $M(0) > 0$, $M(y^*) = 0$, $M(1-\tau) < 0$, there can only be one root for $M(y)$ on the interval $[0,(1-\tau)]$. Therefore, for $\rho < 0$:

$M(y) \gtrless 0$ as $y \lesseqgtr y^*$. Returning to (8):

(8') $$\frac{\partial \overline{U}}{\partial \tau} = H\cdot E\left[\left(\frac{y_\tau}{1-\tau-y}\right) M(y)\ \left\{\left(\frac{1-y}{y}\right)^{\rho-1} \frac{1}{y}\right\}\right] > 0 \text{ since } M(y)y_\tau \geq 0,$$

with strict inequality for $y \neq y^*$.

For $\rho > 0$, $\phi > 0$, $M(0) > 0$, $M(y^*) = 0$, $M(1-\tau) < 0$, and $\frac{d^2M}{dy^2} < 0$; again, this implies $M(y)$ has only one root on $[0,(1-\tau)]$; therefore, $M(y)\cdot y_\tau \geq 0$, and

$M(y)y_\tau > 0$, $y \neq y^*$. Thus, by (8'), $\frac{\partial \overline{U}}{\partial \tau} > 0$.

For $\rho > 0$, $\phi < 0$ another approach is needed. Remember:

(15) $\phi < 0 \to y^* < \hat{y} \to \theta(I-\tau-\hat{y}) < 1$, from (6).

Differentiating (2) with respect to τ and simplifying:

(16) $$\rho\ E\left[A^\rho\left(y^{\rho-1} - \lambda(1-y)^{\rho-1}\right) y_\tau\right] = H\cdot E\left[\left(\frac{1}{1-\tau-y}\right)\left(\lambda\left(\frac{1-y}{y}\right)^{\rho-1} - 1\right)\left(1 + \frac{y_\tau}{y}\,(1-\rho)(1-\tau)\right)\right]$$

Using (3), for any α:

$$(17)\quad \rho\cdot E\left[A^{\rho}\left(y^{\rho-1}-\lambda(1-y)^{\rho-1}\right)y_{\tau}\right] = H\cdot E\left[\left(\frac{1}{1-\tau-y}\right)\left(\lambda\left(\frac{1-y}{y}\right)^{\rho-1}-1\right)\left(1+\frac{y_{\tau}}{y}(1-\rho)(1-\tau)-\alpha(1-\tau-y)\right)\right]$$

Define:

$$(18)\quad K(y,\alpha) = 1 + \frac{y_{\tau}}{y}(1-\rho)(1-\tau) - \alpha(1-\tau-y)$$

Choose $\hat{\alpha}$ s.t. $K(\hat{y}) = 0$, using (5):

$$(19)\quad \hat{\alpha} = \frac{\rho\hat{y} + (1-\rho)(1-\tau)\theta(1-\tau-\hat{y})}{(1-\tau-\hat{y})((1-\rho)(1-\tau) + \rho\hat{y})}$$

$$(20)\quad K(y) = \frac{1}{\left((1-\rho)(1-\tau) + \rho y\right)}\left[\rho y+\theta(1-\rho)(1-\tau)(1-\tau-y)-\hat{\alpha}(1-\tau-y)\left((1-\rho)(1-\tau)+\rho y\right)\right]$$

$$\equiv \frac{K^{*}(y)}{\left((1-\rho)(1-\tau)+\rho y\right)}$$

$$(21)\quad K^{*}(0) = (\theta-\alpha)(1-\rho)(1-\tau)^{2}$$

$$K^{*}(\hat{y}) = 0$$

$$K^{*}(1-\tau) = \rho(1-\tau)$$

$$(22)\quad \frac{d^{2}K^{*}}{dy^{2}} = 2\hat{\alpha}\rho$$

Note that:

$$(23)\quad \theta - \alpha = \frac{\rho\hat{y}[\theta(1-\tau-\hat{y})-1]}{\left((1-\rho)(1-\tau)+\rho\hat{y}\right)(1-\tau-\hat{y})} < 0,\ \rho > 0,\ \hat{y} > y^{*}$$

Consequently, for $\rho > 0$, $\hat{y} > y^{*}$, $K^{*}(0) < 0 = K^{*}(\hat{y}) < K^{*}(1-\tau)$, and $\frac{d^{2}K^{*}}{dy^{2}} > 0$ implies there is only one root to $K^{*}(y)$ on $[0,(1-\tau)]$. Returning to (17):

$$(24)\quad \rho\cdot E\left[A^{\rho}\left(y^{\rho-1}-\lambda(1-y)^{\rho-1}\right)y_{\tau}\right] = H\cdot E\left[\left(\frac{1}{1-\tau-y}\right)\left(\lambda\left(\frac{1-y}{y}\right)^{\rho-1}-1\right)K(y)\right]$$

But $\left[\lambda\left(\frac{1-y}{y}\right)^{\rho-1}-1\right] \lesseqgtr 0$ as $y \lesseqgtr \hat{y}$; $K(y) \lesseqgtr 0$ as $y \lesseqgtr \hat{y}$. So, the product

$K(y)\left[\lambda\left(\frac{1-y}{y}\right)^{\rho-1}-1\right] \geq 0$, with strict equality for $y \neq \hat{y}$.

Hence, $\frac{\partial \bar{U}}{\partial \tau} > 0$ for $\tau < \tau^*$ for all cases. Q.E.D.

SECOND SUPPLEMENT: Taxes and Labor Supply

For the proportional tax scheme, we have seen that the impact of tax increases on labor supply depends only on ρ:

(1) $\frac{\partial L_1}{\partial \tau} \gtreqless 0$ as $\rho \gtreqless 0$.

For the flat rebate tax scheme, from (44):

(2) $\left(\frac{1-L_2}{L_2}\right)^{\rho-1} = (1-\tau)E\left[A^{\rho}y^{\rho-1}\right] = H(\tau)\cdot(1-\tau)\cdot E\left[\frac{1}{(1-\tau-y)}\right]$, using (2) of

Appendix I. Thus,

(3) $(1-\rho)\frac{(1-L_2)^{\rho-2}}{L_2^{\rho}} \; ; \frac{\partial L_2}{\partial \tau} = -H(\tau)E\left[\frac{(\theta(1-\tau)+1)(1-\tau-y)-(1+y_\tau)(1-\tau)}{(1-\tau-y)^2}\right]$

where θ is as defined in (5) of Appendix I.

Substituting for y_τ:

(4) $(1-\rho)\frac{(1-L_2)^{\rho-2}}{(L_2)^{\rho}} \frac{\partial L_2}{\partial \tau} = -H(\tau)E\left[\frac{(1-\rho)(1-\tau)\theta(1-\tau-y)+\rho y}{(1-\tau-y)\left((1-\rho)(1-\tau)+\rho y\right)}\right] < 0, \ \rho > 0.$

Thus, $\frac{\partial L_2}{\partial \tau} < 0$ for $\rho > 0$; for $\rho < 0$ it does not seem possible to demonstrate monotonicity of $\frac{\partial L_2}{\partial \tau}$ (see our simulation results). However, for $\tau = 0$, from (52) of the text:

(5) $(1-\rho)\frac{(1-L_2)^{\rho-2}}{L_2^{\rho}} \frac{\partial L_2}{\partial \tau} = -E\left[A^{\rho}y^{\rho-1}\right] - (1-\tau)(1-\rho)E\left[A^{\rho}y^{\rho-2}y_\tau\right]$;

$$= -E\left[A^{\rho}y^{\rho-1}\right] - (1-\tau)(1-\rho)H(\tau)\cdot E\left[\frac{y_\tau}{y(1-\tau-y)}\right]$$

At $\tau=0$

(6) $E\left[\frac{y_\tau}{y(1-\tau-y)}\right] = E\left[\frac{y_\tau}{y(1-y)}\right] = E\left[\left(y_\tau \frac{(1-y)^{\rho-2}}{y^{\rho}}\right)\left(\frac{y}{1-y}\right)^{\rho-1}\right]$

But $y_\tau \gtreqless 0$ as $y \gtreqless y^*$; and $\left(\frac{y}{1-y}\right)^{\rho-1}$ is decreasing in y; thus:

$$(7)\quad E\left[\frac{y_\tau}{y(1-\tau-y)}\right] = E\left[\left(y_\tau \frac{(1-y)^{\rho-2}}{y^\rho}\right)\left(\frac{y}{1-y}\right)^{\rho-1}\right] > \left(\frac{y^*}{1-y^*}\right)^{\rho-1} E\left[y_\tau \frac{(1-y)^{\rho-2}}{y^\rho}\right] = 0.$$

Consequently, from (5), $\frac{\partial L_2}{\partial \tau} < 0$ for $\tau = 0$.

While we cannot show L_2 decreases monotonically in τ, we have shown $L_2(\tau) < L_1(\tau)$, $\tau > 0$ (L_1 being labor supply for the proportional plan), and $\frac{\partial L_1}{\partial \tau} \lesssim 0$; $\rho < 0$. Thus, the net impact of any tax is to decrease L_2; i.e., $L_2(\tau) < L_2(0)$ for all $\tau > 0$.